3D Steerable Pyramid based on conic filters

Céline A. Delle Luche, Florence Denis and Atilla Baskurt
Lyon Research Center for Images and Intelligent Information Systems
LIRIS FRE 2672 CNRS/INSA de Lyon/Université Claude Bernard Lyon 1
Université Lumière Lyon 2/Ecole Centrale de Lyon
Bat. Nautibus, 43, bd du 11 novembre 1918, F-69622 Villeurbanne Cedex, France

ABSTRACT

The object of this work is 3D directional structures detection. The detection is based on steerable filters, which can be steered to any orientation fixed by the user, and are synthesized using a limited number of basis filters. These filters are used in a recursive multi-scale transform: the steerable pyramid. 2D multiscale approaches using oriented filters have proved to be efficient to detect such curvilinear patterns. We develop a 3D extension of the steerable pyramid to analyze volumes with a desired number of filters.

Keywords: steerable pyramid, directional filter, multiresolution, 3D analysis

1. INTRODUCTION

The concept of steerable filters was first introduced by Freeman and Adelson.[1] The steerable filters are directional derivative operators, which can vary in size and orientation in a way to provide multiscale and multiresolution analysis. They can be recursively applied to successive low-pass bands of the image resulting in a steerable pyramid decomposition.[2]

This kind of pyramid was applied to multiple applications. The first application was the extraction of the contours with respect to a desired direction. This decomposition was used for junction identification in computer vision.[3] It can also be useful for image enhancement[4] and for detecting multidirectional structures (typically stellar shapes) in a noisy background.[5] Moreover we can cite other applications such as texture identification or even denoising.[2]

In this study we will focus on the detection of directional structures in 3D images. So we develop a 3D analysis method based on a multiresolution approach with a 3D conic extension of steerable filters. The difficulty arises when one attempts to create a uniform angular partition of space. The problem has been overcome by aligning the filters directions on the vertices of regular polyhedra. With this solution an invertible transform can be defined, the number of directions under study being limited to 3, 4, 6 or 10. We present interesting results obtained with 3D images.

2. 2D FILTER DESIGN

Figure 1 presents the steerable pyramid decomposition as it was proposed by Simoncelli et al.[2] Initially, the image is separated into low and high-pass subbands. The low-pass image is then divided into k oriented bandpass subbands and a lower-pass subband. This last one is then subsampled by a factor of 2, both in the x and y directions. The recursivity is achieved by inserting another level of decomposition in the lower branch.

Figure 2 shows the frequency decomposition for 3 directions and for 2 resolutions. The bandpass filters B_m must be polar-separable.

Further author information: (Send correspondence to F. Denis)
E-mail: Florence.Denis@liris.univ-lyon1.fr, Phone: +(33) 4 72 43 19 75
Bat. Nautibus, 43, bd du 11 novembre 1918, F-69622 Villeurbanne Cedex, France

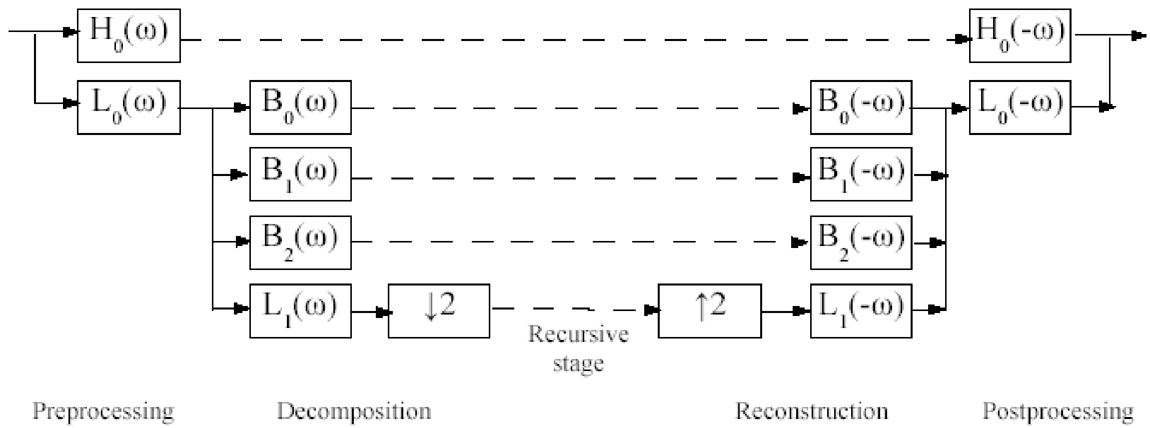

Figure 1. Example of a steerable pyramid, with 3 directions

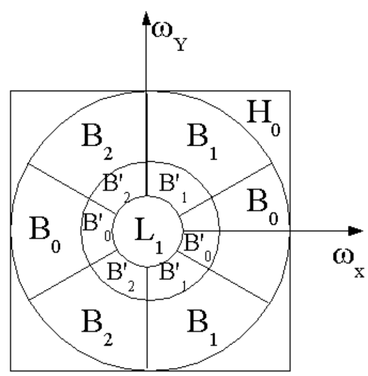

Figure 2. Decomposition in the Fourier domain, for 3 directions and 2 scales

Let ω be the frequency vector in the Fourier domain, f is the radial frequency and θ the angular part. N stands for the chosen number of filters. Therefore $\omega = f.(\cos(\theta)\omega_x + \sin(\theta)\omega_y)$. The algorithm presented on figure 1 for k directions can be described by this equation:

$$R(\omega) = \left(|H_0(\omega)|^2 + |L_0(\omega)|^2 \left(|L_1(\omega)|^2 + \sum_{m=0}^{k-1} |B_m(\omega)|^2 \right) \right) X(\omega) + a.t. \tag{1}$$

$X(\omega)$ stands for the input image in the Fourier domain, $R(\omega)$ for the output image and $a.t.$ for aliasing terms. In order to avoid aliasing and then obtain an invertible transform ($R = X$), some constraints have to be added[6–8]: the low pass filter must be band-limited

$$L_1(\omega) = 0 \text{ for } |\omega| > \frac{\pi}{2} \tag{2}$$

the system must assume unity response:

$$|L_0(\omega)|^2 \left(|L_1(\omega)|^2 + \sum_{k=0}^{N-1} |B_k(\omega)|^2 \right) + |H_0(\omega)|^2 = 1 \tag{3}$$

and the recursion condition must be respected:

$$|L_1(\omega/2)|^2 \left(|L_1(\omega)|^2 + \sum_{k=0}^{N-1} |B_k(\omega)|^2 \right) = |L_1(\omega/2)|^2 \tag{4}$$

The steerable condition for the radial filters can be written as:

$$B_k(\omega) = B(\omega) \left(-i \cos(\theta - \theta_k)\right)^{N-1} \tag{5}$$

with $\theta_k = \pi k/N$.

Satisfying these conditions, Castleman[8] proposed simple high and low-pass filters having cosine transition:

$$HP(a, b, f) = \begin{cases} 0 & , f \leq a \\ \sqrt{\frac{1}{2}\left(1 - \cos\left(\pi(\frac{f-a}{b-a})\right)\right)} & , a < f < b \\ 1 & , f \geq b \end{cases} \tag{6}$$

$$LP(a, b, f) = \begin{cases} 1 & , f \leq a \\ \sqrt{\frac{1}{2}\left(1 + \cos\left(\pi(\frac{f-a}{b-a})\right)\right)} & , a < f < b \\ 0 & , f \geq b \end{cases} \tag{7}$$

and the angular filters have the shape of a raised cosine:

$$B_m(\theta) = HP\left(f_1, \frac{f_N}{2}, f\right) \frac{\left(-i \cos\left(\theta - \frac{m\pi}{N}\right)\right)^{N-1}}{\sqrt{\sum_{n=0}^{N-1} \cos\left(\frac{n\pi}{N}\right)^{2(N-1)}}} \tag{8}$$

Let us note that the denominator in Equation (8) is just a normalization constant so as to assume that $\sum_{m=1}^{N-1} |B_m|^2 + |L_1|^2 = 1$.

3. THE 3D APPROACH

The same idea of pyramidal decomposition with cosine transition applies to 3D images. 3D extension of the low and high-pass filters is easy. But the design of bandpass filters is less trivial, the problem being to distribute the filters on the sphere. Freeman and Adelson[1] proposed a 3D extension of steerable filters based on expansion in series of spherical harmonics. However there exists no uniform shape for the basis functions. As an alternative Andersson[9] proposed to use the regular polyhedra for orienting bandpass filters and then obtain a uniform set of filters on the sphere. Yu et al.[10] have developed a local method based on approximate steerability by using conic gaussian filters. This is a good way for analyzing junctions but it is not a global method.

We draw from both Andersson and Yu methods. We want to tessellate the Fourier space in order to build a pyramid. The only way to obtain a regular mapping of the filters is to use the regular polyhedra. Then we choose to compute our filters thanks to the directions of the vertices of the polyhedra but we have to respect the filter conditions cited by Castleman.[8] The most difficult thing is to obtain : $\sum_{m=1}^{N-1} |B_m|^2 + |L_1|^2 = 1$.

By using the regular polyhedra we developed an invertible transform whose conic bandpass filters can be written:

$$B_m(\alpha) = \cos(\alpha)^n \tag{9}$$

α is the angle between the center point C and a point M. It defines a cone, as shown in figure 3 and can be obtained:

$$\cos(\alpha) = \frac{\overrightarrow{OM}.\overrightarrow{OC}}{\|OM\|\|OC\|} \tag{10}$$

We prove that for an octahedron or a cube(3 or 4 directions) $n = 1$ and for an icosahedron or a dodecahedron (6 or 10 directions) $n = 2$. As a result we also can use the compound of a dodecahedron and an icosahedron (the truncated icosahedron, much known as the soccer ball) by linear composition, we obtain a 16-direction filter. Figure 5 shows the repartition for a 6-direction filter and Figure 4 represents the radial transition on the filters. The filters intersect at 2.7 dB, so this is almost a 3 dB cut-off frequency. Here the filter no. 6 is oriented through the z axis, but by simple rotation the axes can be adjusted as wanted.

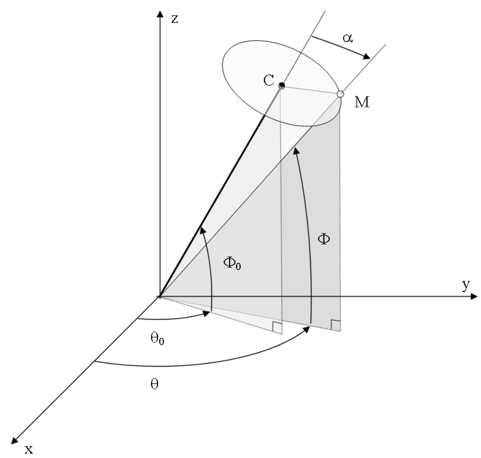

Figure 3. Definition of our filter in spherical coordinates

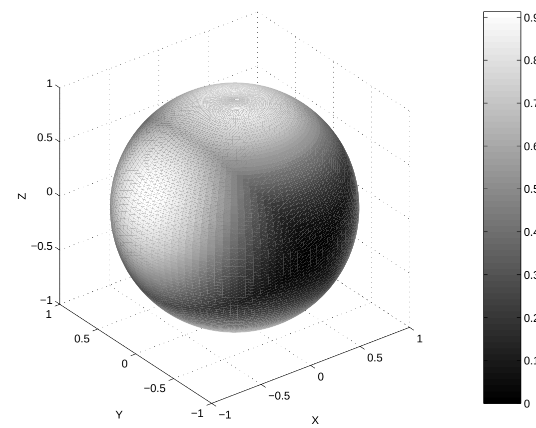

Figure 4. Transition between two filters on the sphere (for a 6 directions-filter)

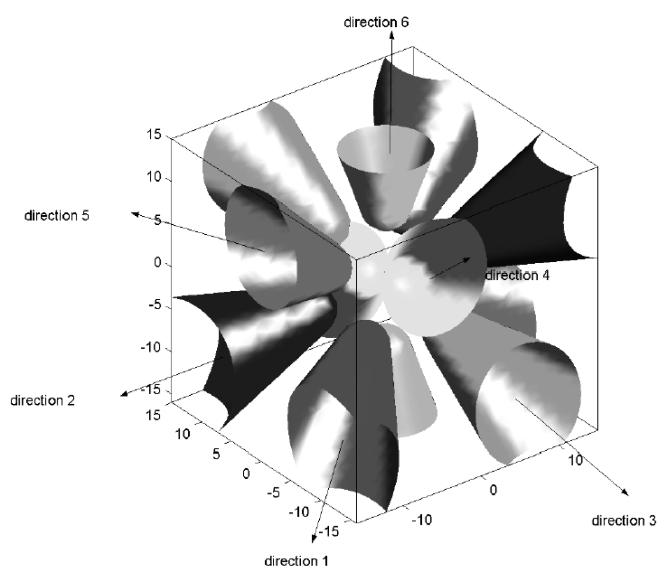

Figure 5. Filters for 6 directions, represented as isosurfaces

4. APPLICATIONS

Several domains of application can be considered when using such pyramids, especially those associated to the enhancement and detection of directional structures in industrial vision and in medical imaging; multidirectional and multiresolution analysis for 3D data denoising and for data compression. These pyramids can also be used for video content analysis in order to detect global motions as a function of the time (if we consider some privileged directions through the time axis. They extract the contours through the orthogonal direction of the filters.

For example, our filter bank was applied on a volumic image (120 x 68 x 64 voxels, 256 gray levels) represented as an isosurface and few slices in figure 6. It is remarkable in figure 7 that the volume is mainly oriented in the direction of the third subband, so we guess that the response in the subband no. 3 will be less important.

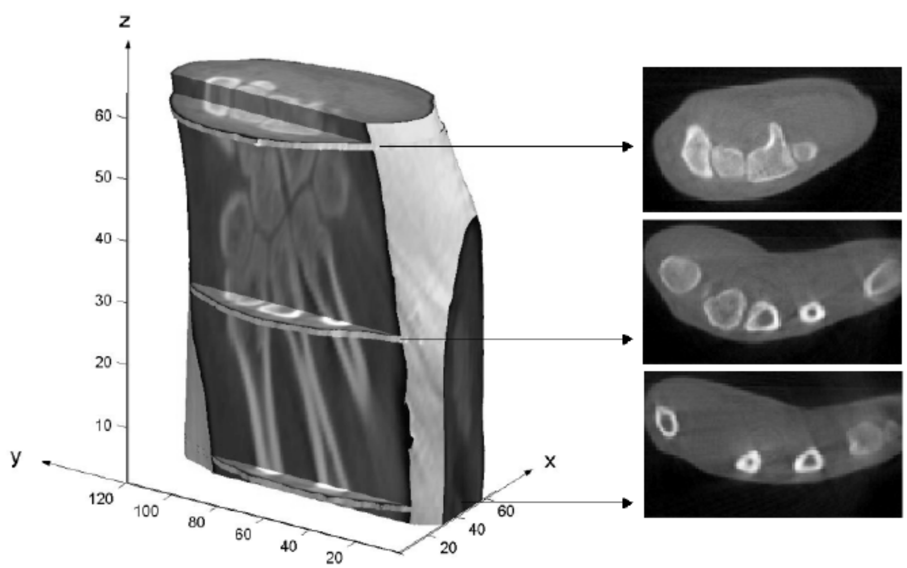

Figure 6. Original image, isosurface and slices (no. 10, 30 and 60) in z axis

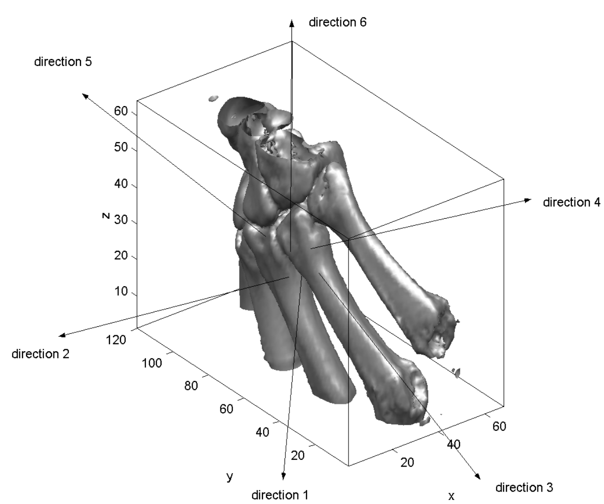

Figure 7. Original image, with the directions of the chosen filters

We can observe the results of the filtered volume through 6 directions. The following pictures (Fig. 8 to 10) show the results of the 6 direction filter by displaying the projections (maximum absolute value of the filtered volume along x, y, and z direction) through the different axes. As it was said before, the filtered data in subband no. 3 is faint, because the principal orientation of the fingers is in the third filter direction. The fingers at the right hand of the picture are not well visible. But in the subband no. 4, which is almost orthogonal to these fingers, the contours are clearly detected. In the same way, the skin which is oriented in the z axis, does not appear in the subband no. 6.

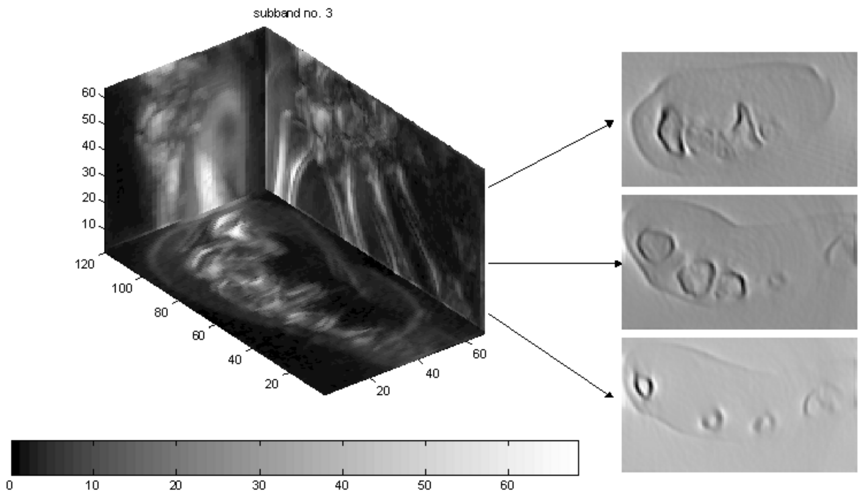

Figure 8. Subband no. 3, with slides no. 10, 30 and 60

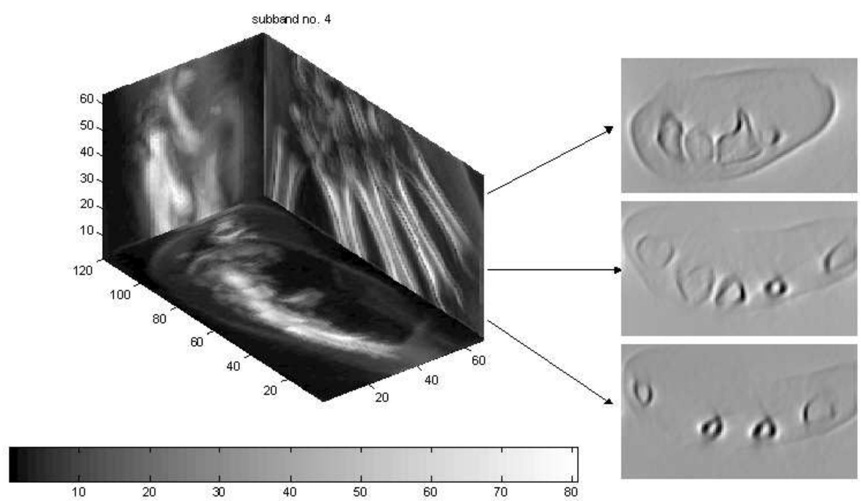

Figure 9. Subband no. 4, with slides no. 10, 30 and 60

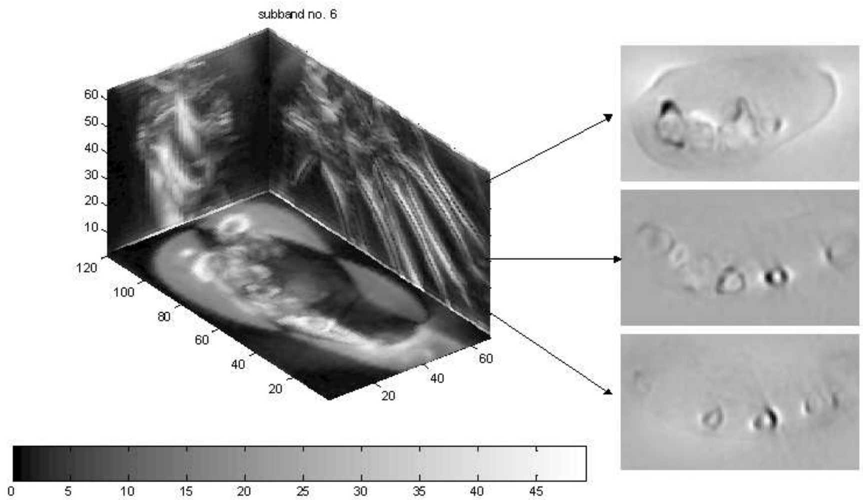

Figure 10. Subband no. 6, with slides no. 10, 30 and 60

An other application can be the analysis of video. Here is an example of a small video (173 x 230 x 104), Figures 11, 12, 13 represent some slides of this video. It is quite obvious that there is a motionless background and two people moving, essentially in the direction of the columns (y axis).

Figure 11. Video : image no. 1, z is the temporal axis

Figure 12. Image no. 50

Figure 13. Image no. 100

The video was filtered with a 6-direction filter, and let us observe the subband directed with the z axis. The following picture (Figure 14) shows the projections on the different axes. This subband highlights the contours that vary during the time. In the temporal projection, the global motion of the couple is clearly detected and we can see that there is a quite important amount of motion in the time axis. The x projection shows the movement through the y direction : we see the contours moving to the left (y decreasing) with the time. And the projection in the line direction displays the fact that the motion in x direction is not important.

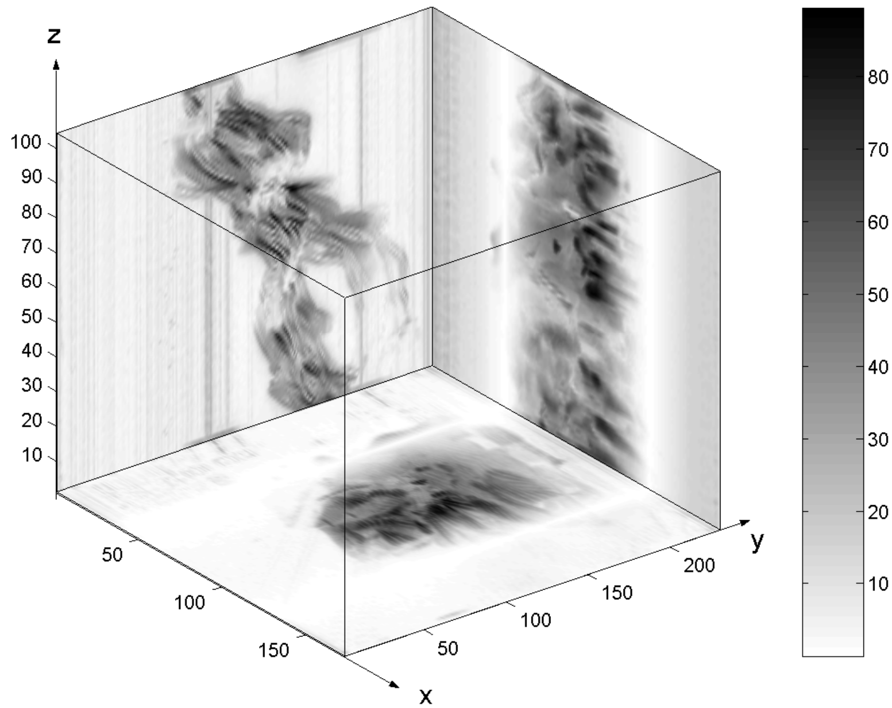

Figure 14. Projections of the subband no. 6 according to the different axes

5. CONCLUSION

We presented a 3D steerable pyramid based on conic filters. This is a powerful tool for volume and video analysis, thanks to the multiresolution decomposition and the perfect reconstruction. This new tool is a good candidate for 3D data analysis when directional variations in gray levels, color, multispectral data and also in time (video sequences), have to be detected.

The improvement of the angular selectivity in order to perform a finer analysis, is under investigation.

ACKNOWLEDGMENTS

We would like to thank Pr. Jean-Marie Becker, from LISA (CPE Lyon), for his help in mathematics.

REFERENCES

1. W. T. Freeman and E. Adelson, "The design and use of steerable filters," *IEEE Trans. Patt. Anal. Machine Intell.* **13**, pp. 891–906, September 1991.
2. E. Simoncelli, W. Freeman, E. Adelson, and D. J. Heeger, "Shiftable multiscale transforms," *IEEE Trans. Information Theory* **38**, pp. 587–607, March 1992.
3. E. Simoncelli and H. Farid, "Steerable wedge filters for local orientation analysis," *IEEE Trans. Image Processing* **5**, pp. 1377–1382, September 1996.
4. Q. Wu, M. A. Schulze, and K. R. Castleman, "Steerable pyramid filters for selective image enhancement applications," *Proc. ISCAS'98*, 1998.
5. F. Denis, F. Davignon, and A. Baskurt, "Steerable pyramid for contrast enhancement and directional structures detection," *XI European Signal Processing Conference (EUSIPCO 2002), Toulouse*, 3-6 September 2002.
6. E. P. Simoncelli and W. T. Freeman, "The steerable pyramid: a flexible architecture for multiscale derivative computation," *2nd IEEE International Conference on Image Processing*, pp. 444–447, October 1995.

7. A. Karasaridis and E. P. Simoncelli, "A filter design technique for steerable pyramid image transforms," *Proc. IEEE ICASSP*, May 1996.
8. K. R. Castleman, M. A. Schulze, and Q. Wu, "Simplified design of steerable pyramid filters," *Proc. ISCAS'98*, 1998.
9. M. T. Andersson, *Controllable Multi-dimensional Filters and Models in Low-Level Computer Vision.* PhD thesis, Linkoeping University, Sweden, 1992.
10. W. Yu, K. Daniilidis, and G. Sommer, "A new 3D orientation steerable filter," *DAGM Symposium Mustererkennung, Kiel*, pp. 203–212, September 2000.